Description of the benchmark examples in *COMPl$_e$ib* 1.0

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Abstract. In this note, we state a short description of each individual test example which is contained in *COMPl$_e$ib* 1.0 – the *CO*nstrained *M*atrix–optimization *P*roblem *lib*rary [37], [38]. The problems are drawn from a variety of control systems engineering applications. The current version of *COMPl$_e$ib* contains 124 problems coded in standard MATLAB matrix format. The advantage of this format is the platform independence. The data of the currently available test examples of *COMPl$_e$ib* 1.0 are collected in the MATLAB file *COMPleib.m* which is available over the internet. This file contains a single MATLAB function which returns the data of the individual examples. For more details, we refer the interested reader to the user guide of *COMPl$_e$ib* [38] and the companion paper [37]. In particular, as discussed in [37], the test examples in *COMPl$_e$ib* 1.0 can be used as a benchmark collection for a very wide variety of algorithms solving (constraint) matrix optimization problems. For example, *COMPl$_e$ib* can be used for testing solvers for nonlinear semidefinite programs (NSDPs), bilinear matrix inequality (BMI) problems, linear semidefinite programs (SDPs), Riccati or Lyapunov equations and other related matrix problems. An incomplete list of such problems is given in [37] which is far from being exhaustive.

Key Words. *collection of test examples; constrained matrix optimization–problem library*

AMS subject classification.

1. Data structure of *COMPl$_e$ib* . As described in [37], [38], release 1.0 of the constrained matrix optimization problem library *COMPl$_e$ib* consists of more than 120 examples collected from the engineering literature and real–life (engineering) applications for LTI control systems. In example, consider a LTI plant of order n_x with state space realization:

$$\begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B_1 w(t) + Bu(t), \\ z(t) & = & C_1 x(t) + D_{11} w(t) + D_{12} u(t), \\ y(t) & = & Cx(t) + D_{21} w(t), \end{array} \tag{1.1}$$

where $x \in \boldsymbol{R}^{n_x}$, $u \in \boldsymbol{R}^{n_u}$, $y \in \boldsymbol{R}^{n_y}$, $z \in \boldsymbol{R}^{n_z}$, $w \in \boldsymbol{R}^{n_w}$ denote the state, control input, measured output, regulated output, and noise input, respectively. The current version of *COMPl$_e$ib* consists simply of the data matrices $A \in \boldsymbol{R}^{n_x \times n_x}$, $B_1 \in \boldsymbol{R}^{n_x \times n_w}$, $B \in \boldsymbol{R}^{n_x \times n_u}$, $C_1 \in \boldsymbol{R}^{n_z \times n_x}$, $D_{11} \in \boldsymbol{R}^{n_z \times n_w}$, $D_{12} \in \boldsymbol{R}^{n_z \times n_u}$, $C \in \boldsymbol{R}^{n_y \times n_x}$ and $D_{21} \in \boldsymbol{R}^{n_x \times n_w}$. In particular, all 124 test problems in *COMPl$_e$ib* 1.0 are coded and stored in standard MATLAB matrix format. We have decided to use this format, since the main advantage of MATLAB is the platform independence. The heart of *COMPl$_e$ib* is the MATLAB function file *COMPleib.m*. In a MATLAB environment, the data of the individual test example of *COMPl$_e$ib* can be accessed by calling the MATLAB function therein. For example, in MATLAB, the command

>> [A,B1,B,C1,C,D11,D12,D21,nx,nw,nu,nz,ny] = COMPleib('AC1');

returns the real data matrices A, B_1, B, C_1, C, D_{11}, D_{12} and D_{21} of (1.1) as well as the integers (dimension parameters) n_x, n_w, n_u, n_z and n_y of the *COMPl$_e$ib* example *AC1*. Together with the MATLAB function file *COMPleib.m*, *COMPl$_e$ib* is provided with several binary MATLAB data files (MAT-files) which contains the data matrices of some individual (large) test examples. In particular, release 1.0 of *COMPl$_e$ib* contains also the following MAT-files: *ac10.mat, ac13_14.mat, ac18.mat, bdt2.mat, cbm.mat, cdp.mat, cm1.mat – cm6.mat (6 files), dlr2_3.mat, he6.mat, he7.mat, hf2d1.mat – hf2d18.mat (18 files), ih.mat, iss1_2.mat, je1.mat, je2_3.mat, lah.mat, tl.mat.* Note, the name of the MAT-file corresponds to the example name in *COMPl$_e$ib* . For more details we refer to the *COMPl$_e$ib* user manual [38].

2. Benchmark examples in *COMPl$_e$ib* . In this section we give a short overview of each individual test example which is implemented in *COMPl$_e$ib* 1.0. Moreover, we state some detailed information about the source and the application (if any) of the *COMPl$_e$ib* 1.0 benchmark problems.

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At the current stage, *COMPl$_e$ib* is divided into problem sets and the problem sets are grouped into problem classes. The problem classes and sets are listed in the tables and paragraphs below.

Table 2.1 provides a list of the first problem class. In this class are those examples which are static output feedback (SOF) stabilizable (i. e. see [37] and the references therein). More detailed information of each example in this class are given in Subsection 2.1.

Table 2.1: *Static output feedback control examples*

Example	n_x	n_u	n_y	Structure of A	Example	n_x	n_u	n_y	Structure of A
(AC1)	5	3	3	dense	(WEC3)	10	3	4	dense
(AC2)	5	3	3	dense	(HF1)	130	1	2	sparse
(AC3)	5	2	4	dense	(BDT1)	11	3	3	sparse
(AC4)	4	1	2	dense	(BDT2)	82	4	4	sparse
(AC5)	4	2	2	dense	(MFP)	4	3	2	dense
(AC6)	7	2	4	dense	(UWV)	8	2	2	dense
(AC7)	9	1	2	dense	(IH)	21	11	10	sparse
(AC8)	9	1	5	dense	(CSE1)	20	2	10	sparse
(AC9)	10	4	5	dense	(CSE2)	60	2	30	sparse
(AC10)	55	2	2	sparse	(EB1)	10	1	1	sparse
(AC11)	5	2	4	dense	(EB2)	10	1	1	sparse
(AC12)	4	3	4	dense	(EB3)	10	1	1	sparse
(AC13)	28	3	4	sparse	(EB4)	20	1	1	sparse
(AC14)	40	3	4	sparse	(EB5)	40	1	1	sparse
(AC15)	4	2	3	dense	(EB6)	160	1	1	sparse
(AC16)	4	2	4	dense	(PAS)	5	1	3	dense
(AC17)	4	1	2	dense	(TF1)	7	2	4	dense
(AC18)	10	2	2	dense	(TF2)	7	2	3	dense
(HE1)	4	2	1	dense	(TF3)	7	2	3	dense
(HE2)	4	2	2	dense	(PSM)	7	2	3	dense
(HE3)	8	4	6	dense	(TL)	256	2	2	dense
(HE4)	8	4	6	dense	(CDP)	120	2	2	sparse
(HE5)	8	4	2	dense	(NN1)	3	1	2	dense
(HE6)	20	4	6	dense	(NN2)	2	1	1	dense
(HE7)	20	4	6	dense	(NN3)	4	1	1	dense
(JE1)	30	3	5	partly sparse	(NN4)	4	2	3	dense
(JE2)	21	3	3	dense	(NN5)	7	1	2	dense
(JE3)	24	3	6	dense	(NN6)	9	1	4	dense
(REA1)	4	2	3	dense	(NN7)	9	1	4	dense
(REA2)	4	2	2	dense	(NN8)	3	2	2	dense
(REA3)	12	1	3	dense	(NN9)	5	3	2	dense
(REA4)	8	1	1	dense	(NN10)	8	3	3	dense
(DIS1)	8	4	4	dense	(NN11)	16	3	5	dense
(DIS2)	3	2	2	dense	(NN12)	6	2	2	dense
(DIS3)	6	4	4	dense	(NN13)	6	2	2	dense
(DIS4)	6	4	6	dense	(NN14)	6	2	2	dense
(DIS5)	4	2	2	dense	(NN15)	3	2	2	dense
(TG1)	10	2	2	dense	(NN16)	8	4	4	dense
(AGS)	12	2	2	sparse	(NN17)	3	2	1	dense
(WEC1)	10	3	4	dense	(NN18)	1006	1	1	sparse
(WEC2)	10	3	4	dense					

Note, we subdivide this *COMPl$_e$ib* class into the following problem sets:

- Aircraft models *(AC)*
- Helicopter models *(HE)*
- Jet engine models *(JE)*
- Reactor models *(REA)*
- Decentralized interconnected systems *(DIS)*
- Euler Bernoulli beams *(EB)*
- Academic test problems *(NN)*.

Moreover, some further examples from different applications in this class are, i. e. , wind energy conversion models *(WEC)*, binary distillation towers *(BDT)*, terrain following models *(TF)*, and a compact disk player *(CDP)*.

Another group of *COMPleib* examples can be found in Table 2.2. These problems arise in the design of (static) output feedback control laws for two dimensional heat flow models. The original models are infinite dimensional control problems. Using a suitable discretization scheme yields a corresponding (in general) large scale finite dimensional approximation of the original problem. A detailed discussion and some case studies of the 2D heat flow models in *COMPleib* is given in [37, Section 3] (see also the short discussion in Paragraph 2.2). The first nine examples represent the approximation of the discretized 2D heat flow models, while the other nine are the corresponding highly reduced order approximations of the large dimensional systems gained by the proper orthogonal decomposition (POD) approach as discussed in [40].

TABLE 2.2
2D heat flow models [37, Section 3]

Large model (sparse)					POD model (dense)					Property of	
Example	n_x	n_u	n_y	δ	Example	n_x	n_u	n_y	δ	A	model
(HF2D1)	3796	2	3	0.3825	(HF2D10)	5	2	3	0.3825	unstable	nonlinear
(HF2D2)	3796	2	3	0.5325	(HF2D11)	5	2	3	0.5325	unstable	nonlinear
(HF2D3)	4489	2	4	0	(HF2D12)	5	2	4	0	stable	linear
(HF2D4)	2025	2	4	0	(HF2D13)	5	2	4	0	stable	linear
(HF2D5)	4489	2	4	0.3825	(HF2D14)	5	2	4	0.3825	unstable	linear
(HF2D6)	2025	2	4	1.725	(HF2D15)	5	2	4	1.725	unstable	linear
(HF2D7)	4489	2	4	0.2775	(HF2D16)	5	2	4	0.2775	unstable	nonlinear
(HF2D8)	2025	2	4	0.7575	(HF2D17)	5	2	4	0.7575	unstable	nonlinear
(HF2D9)	3481	2	2	0.47813	(HF2D18)	5	2	2	0.47813	unstable	linear

The examples listed in Table 2.3 represent so–called second order models which can be rewritten into first order ODEs (see Subsection 2.3). Note, in this case, the system matrices have a special structure. This is the reason why we have collected those problems in an extra class. But, note, all currently *COMPleib* examples in this problem class are also SOF stabilizable which, in general, is not always true for second order models.

TABLE 2.3
Second order models

Example	n_x	n_u	n_y	Structure of A	Example	n_x	n_u	n_y	Structure of A
(CM1)	20	1	2	partly sparse	(DLR1)	10	2	2	dense
(CM2)	60	1	2	partly sparse	(DLR2)	40	2	2	sparse
(CM3)	120	1	2	partly sparse	(DLR3)	40	2	2	sparse
(CM4)	240	1	2	partly sparse	(ISS1)	270	3	3	sparse
(CM5)	480	1	2	partly sparse	(ISS2)	270	3	3	sparse
(CM6)	960	1	2	partly sparse	(CBM)	348	1	1	partly sparse
(TMD)	6	2	4	dense	(LAH)	48	1	1	partly sparse
(FS)	5	1	3	dense					

Briefly this class is divided into the following problem sets:

- six so–called cable mass models with very low damping *(CM)*
- three models of a space structure developed by the "Deutsche Forschungsanstalt für Luft- und Raumfahrt" *(DLR)*
- two instances of a component of the International Space Station *(ISS)*
- some other second order models, i. e. a tuned mass damper *(TMD)* example, a clamped beam model *(CBM)*, a flexible satellite *(FS)* example, and, finally, a model of the Los Angeles (university) hospital *(LAH)*

The last class of test examples in *COMPleib* 1.0 are so–called reduced order control (ROC) problems. These instances are not SOF stabilizable, but they are at least stabilizable by a reduced order output feedback control law of order n_c. Table 2.4 gives an overview of the currently implemented ROC problems (see also Paragraph 2.4). Therein, n_c denotes the smallest possible order of the reduced output feedback controller which can be used for stabilizing the control system.

For more details on ROC problems, we refer the interested reader to [37], [40] and the references therein.

TABLE 2.4
Reduced order control instances

Example	n_x	n_u	n_y	n_c	Structure of A	Example	n_x	n_u	n_y	n_c	Structure of A
(ROC1)	9	2	2	1	dense	(ROC6)	5	3	3	2	dense
(ROC2)	10	2	3	1	dense	(ROC7)	5	2	3	1	dense
(ROC3)	11	4	4	2	dense	(ROC8)	9	4	4	3	dense
(ROC4)	9	2	2	1	dense	(ROC9)	6	3	3	2	dense
(ROC5)	7	3	5	1	dense	(ROC10)	6	2	4	1	dense

2.1. Static output feedback control examples.

2.1.1. Aircraft models.

(AC1). This system is a variation of the one described by Y. S. Hung and A. G. J. MacFarlane. It is a state–space model of the linearized vertical–plane dynamics of an aircraft with three inputs, three outputs and five states. For further details see [34], p. 137/169.

$$A = \begin{bmatrix} 0 & 0 & 1.132 & 0 & -1 \\ 0 & -0.0538 & -0.1712 & 0 & 0.0705 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0.0485 & 0 & -0.8556 & -1.013 \\ 0 & -0.2909 & 0 & 1.0532 & -0.6859 \end{bmatrix}, B_1 = \begin{bmatrix} 0.03593 & 0 & 0.01672 \\ 0 & 0.00989 & 0 \\ 0 & -0.07548 & 0 \\ 0 & 0 & 0.05635 \\ 0.00145 & 0 & 0.06743 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ -0.12 & 1 & 0 \\ 0 & 0 & 0 \\ 4.419 & 0 & -1.665 \\ 1.575 & 0 & -0.0732 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(AC2). Like *(AC1)* with changes made in C_1, D_{11} and D_{12}:

$$C_1 = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix},$$

(AC3). A lateral axis model of an L-1011 aircraft in cruise flight conditions presented by C. Edwards and S. K. Spurgeon. For information about state, input and output vectors see [18].

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & -0.154 & -0.0042 & 1.54 & 0 \\ 0 & 0.249 & -1 & -5.2 & 0 \\ 0.0386 & -0.996 & -0.0003 & -0.117 & 0 \\ 0 & 0.5 & 0 & 0 & -0.5 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -0.744 & -0.032 \\ 0.337 & -1.12 \\ 0.02 & 0 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(AC4). B. Fares, P. Apkarian and D. Noll presented the following autopilot control problem for an air–to–air missile [20].

$$A = \begin{bmatrix} -0.876 & 1 & -0.1209 & 0 \\ 8.9117 & 0 & -130.75 & 0 \\ 0 & 0 & -150 & 0 \\ -1 & 0 & 0 & -0.05 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 150 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.25 & 0 & 0 & 3.487 \\ 0 & 0 & -3 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0.25 \\ 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \\ 3 \end{bmatrix},$$

$$C = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & 1 \\ 0.01 & 0 \end{bmatrix}$$

(AC5). A two–input, two–output, fourth–order plant which describes the motion of a Boeing B-747 aircraft flying at an altitude of 20,000 ft with a speed of Mach 0.8 [35].

$$A = \begin{bmatrix} 0.9801 & 0.0003 & -0.0980 & 0.0038 \\ -0.3868 & 0.9071 & 0.0471 & -0.0008 \\ 0.1591 & -0.0015 & 0.9691 & 0.0003 \\ -0.0198 & 0.0958 & 0.0021 & 1 \end{bmatrix}, \; B = \begin{bmatrix} -0.0001 & 0.0058 \\ 0.0296 & 0.0153 \\ 0.0012 & -0.0908 \\ 0.0015 & 0.0008 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(AC6). An L-1011 aircraft model.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -0.154 & -0.0042 & 1.54 & 0 & -0.744 & -0.032 \\ 0 & 0.249 & -1 & -5.2 & 0 & 0.337 & -1.12 \\ 0.0386 & -0.996 & -0.0003 & -2.117 & 0 & 0.02 & 0 \\ 0 & 0.5 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -25 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 25 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & -0.154 & -0.0042 & 1.54 & 0 & -0.744 & -0.032 \\ 0 & 0.249 & -1 & -5.2 & 0 & 0.337 & -1.12 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(AC7). A model for a modern transport airplane at an altitude of 35,000 ft, with a speed of Mach 0.57 and with the center of gravity at the most aft location. The data with further details on the airplane and the flight conditions can be found in [26], Case study III 2), p.1006/1013.

$$A = \begin{bmatrix} -0.06254 & 0.01888 & 0 & -0.56141 & -0.02751 & 0 & 0.06254 & -0.00123 & 0 \\ 0.01089 & -0.9928 & 0.99795 & 0.00097 & -0.07057 & 0 & -0.01089 & 0.06449 & 0 \\ 0.07743 & 1.6754 & -1.31111 & -0.0003 & -4.2503 & 0 & -0.07743 & -0.10883 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -30 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.88206 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.88206 & 0.00882 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.00882 & -0.88206 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.3282 & 0 & 0 & 0 \\ 0 & 1.62671 & 0 & 0 \\ 0 & -68.75283 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 30 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} -0.00519 & 0.47604 & 0.00098 & -0.00031 & 0.03378 & 0 & 0.00519 & -0.03086 & 0 \end{bmatrix}, \; D_{11} = 0_{1\times 4}, \; D_{12} = \begin{bmatrix} \frac{1}{\sqrt{2}} \end{bmatrix},$$

$$C = \begin{bmatrix} -0.00519 & 0.47604 & 0.00098 & -0.00031 & 0.03378 & 0 & 0.00519 & -0.03086 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(AC8). This model describes the same aircraft as in *(AC7)* at an altitude of 35,000 ft, with a speed of Mach 0.8 and with the center of gravity at an aft location (less than in *(AC7)*). For more information see again [26], Case study II, p.1001/1012.

$$A = \begin{bmatrix} -0.01365 & 0.178 & 0.00017 & -0.561 & -0.03726 & 0 & 0.01365 & -0.01311 & 0 \\ -0.01516 & -0.752 & 1.001 & 0.00127 & -0.06311 & 0 & 0.01516 & 0.05536 & 0 \\ 0.00107 & 0.07896 & -0.8725 & 0 & -3.399 & 0 & -0.00107 & -0.00581 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & 10.72 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -50 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.4447 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4447 & 0.0044 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0044 & -0.4447 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 0 \\ 0.9431 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.155 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -48.82 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 50 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.0032 & 0.16015 & -0.01679 & 0 & -0.0516 & 0 & -0.00323 & -0.01179 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & -0.5 & 0 & 0 \end{bmatrix}, \quad D_{11} = 0_{2\times 10}, \quad D_{12} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.0064 & 0.3203 & -0.03358 & 0 & -0.1032 & 0 & -0.00646 & -0.02358 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ -0.01365 & 0.178 & 0.00017 & -0.561 & -0.03726 & 0 & 0.01365 & -0.01311 & 0 \\ 0 & -13.58 & 0 & 13.58 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; D_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(AC9). A variation of *(AC8)* with one more state and four control variables instead of one.

$$A = \begin{bmatrix} -0.01365 & 0.178 & 0.00017 & -0.561 & -0.03726 & 0 & 0.01365 & -0.01311 & 0 & -1 \\ -0.01516 & -0.752 & 1.001 & 0.00127 & -0.06311 & 0 & 0.01516 & 0.05536 & 0 & 0 \\ 0.00107 & 0.07896 & -0.8725 & 0 & -3.399 & 0 & -0.00107 & -0.00581 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & 10.72 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -50 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.4447 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4447 & 0.0044 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0044 & -0.4447 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 & 0 \\ 0.9431 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.155 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -48.82 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 50 & 0 & 0 & 0 \\ 0 & 0.9431 & 0 & 0 \\ 0 & 0 & 1.155 & 0 \\ 0 & 0 & -48.82 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.00646 & 0.3203 & -0.03358 & 0 & -0.1032 & 0 & -0.00646 & -0.02358 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}, \quad D_{11} = 0_{2\times 10}, \quad D_{12} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.00646 & 0.3203 & -0.03358 & 0 & -0.1032 & 0 & -0.00646 & -0.02358 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ -0.01365 & 0.178 & 0.00017 & -0.561 & -0.03726 & 0 & 0.01365 & -0.01311 & 0 & 0 \\ 0 & -13.58 & 0 & 13.58 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; D_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(AC10). This plant is an aeroelastic model of high dynamic order (55th–order) with two inputs and two outputs. It describes a modified Boeing B-767 airplane, at a flutter condition. A detailed formulation can be found in [14]. The data–matrices A, B_1, B, C_1, D_{12}, C and D_{21} are provided in file *"ac10.mat"*, while D_{11} is set 0.

(AC11). This linearized model of an CCV-type aircraft appears in [2].

$$A = \begin{bmatrix} -1.341 & 0.9933 & 0 & -0.1689 & -0.2518 \\ 43.223 & -0.8693 & 0 & -17.251 & -1.5766 \\ 1.341 & 0.0067 & 0 & 0.1689 & 0.2518 \\ 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & -20 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 20 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 47.76 & -0.268 & 0 & -4.56 & 4.45 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(AC12). The linearized equations of motion for the longitudinal dynamics of an Advanced Short Take–Off and Vertical Landing (ASTOVL) aircraft, valid at low speeds in the transition zone from jet–borne to fully wing–borne flight [58].

$$A = \begin{bmatrix} -0.0017 & 0.0413 & -5.3257 & -9.7565 \\ -0.0721 & -0.3393 & 49.5146 & -1.0097 \\ -0.0008 & 0.0138 & -0.2032 & 0.0009 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0.033 & 0 & 0 \\ 0 & 0.048 & -0.002 \\ -0.064 & 0 & 0.340 \\ 0 & 0 & 0.006 \end{bmatrix}, \ B = \begin{bmatrix} 0.2086 & -0.0005 & -0.0271 \\ -0.0005 & 0.2046 & 0.0139 \\ -0.0047 & 0.0023 & 0.1226 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 57.2958 & 0 \\ 0 & 0 & 0 & 57.2958 \\ 0.1045 & -0.9945 & 0.1375 & 51.5791 \\ -0.0002 & 0.0045 & 0 & 0 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.0212 & 0 & 0 \end{bmatrix}$$

(AC13). An augmented version of the ASTOVL aircraft model *(AC12)* of order 28. The data matrices A, B and C are provided in file *"ac13_14.mat"*.

(AC14). An augmented version of the ASTOVL aircraft model *(AC12)* of order 40. The complete data set consisting of A, B_1, B, C_1, D_{11}, D_{12}, C and D_{21} is given in file *"ac13_14.mat"*.

(AC15). This numerical data refers to a Mach 2.7 flight condition of a supersonic transport aircraft. It was given by S. S. Choi and H. R. Sirisena [13].

$$A = \begin{bmatrix} -0.037 & 0.0123 & 0.00055 & -1 \\ 0 & 0 & 1 & 0 \\ -6.37 & 0 & -0.23 & 0.0618 \\ 1.25 & 0 & 0.016 & -0.0457 \end{bmatrix}, \qquad B_1 = I_4, \qquad B = \begin{bmatrix} 0.00084 & 0.000236 \\ 0 & 0 \\ 0.08 & 0.804 \\ -0.0862 & -0.0665 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} I_4 \\ 0_{2\times 4} \end{bmatrix}, \qquad D_{11} = 0_{6\times 4}, \qquad D_{12} = \begin{bmatrix} 0_{4\times 2} \\ I_2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad D_{21} = 0_{3\times 4}$$

(AC16). A state feedback version of *(AC15)* with changed matrices C and D_{21}:

$$C = I_4, \qquad D_{21} = 0_{4\times 4}$$

(AC17). This example consists of a model of the lateral axis dynamic for a L-1011 aircraft. For additional details see [25].

$$A = \begin{bmatrix} -2.98 & 0.93 & 0 & -0.034 \\ -0.99 & -0.21 & 0.035 & -0.0011 \\ 0 & 0 & 0 & 1 \\ 0.39 & -5.555 & 0 & -1.89 \end{bmatrix}, \quad B = \begin{bmatrix} -0.032 \\ 0 \\ 0 \\ -1.6 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(AC18). Here we present a reduced system generated from the data given in *(AC10)*. The system has now an order of ten and again two inputs and two outputs. File *"ac18.mat"* contains the matrices A, B_1, B, C_1, D_{12}, C and D_{21}, while D_{11} is again set to 0.

2.1.2. Helicopter models.

(HE1). The following model of the longitudinal motion of a VTOL helicopter for typical loading and flight condition at the airspeed of 135 knots was introduced by S. N. Singh and A. A. R. Coelho in [55]. (See also [36].)

$$A = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.42 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.04678 & 0 \\ 0.04572 & 0.00988 \\ 0.04369 & 0.00111 \\ -0.02179 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

(HE2). These system matrices describe the longitudinal–vertical motion of an AH-64 helicopter at 130 knots. The model was borrowed from [21].

$$A = \begin{bmatrix} -0.0649 & 0.0787 & 0.1705 & -0.5616 \\ 0.0386 & -0.939 & 4.2277 & 0.0198 \\ 0.1121 & -0.4254 & -0.7968 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -0.9454 & 0.5313 \\ -8.6476 & -10.769 \\ 19.0824 & -2.8959 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(HE3). In this example a Bell 201A-1 helicopter is considered. The linearized helicopter dynamics are described by an eight order, four input, six output state space model. For details see [29], p. 26.

$$A = \begin{bmatrix} -0.0046 & 0.038 & 0.3259 & -0.0045 & -0.402 & -0.073 & -9.81 & 0 \\ -0.1978 & -0.5667 & 0.357 & -0.0378 & -0.2149 & 0.5683 & 0 & 0 \\ 0.0039 & -0.0029 & -0.2947 & 0.007 & 0.2266 & 0.0148 & 0 & 0 \\ 0.0133 & -0.0014 & -0.4076 & -0.0654 & -0.4093 & 0.2674 & 0 & 9.81 \\ 0.0127 & -0.01 & -0.8152 & -0.0397 & -0.821 & 0.1442 & 0 & 0 \\ -0.0285 & -0.0232 & 0.1064 & 0.0709 & -0.2786 & -0.7396 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.0676 \\ -1.1151 \\ 0.0062 \\ -0.017 \\ -0.0129 \\ 0.139 \\ 0 \\ 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0676 & 0.1221 & -0.0001 & -0.0016 \\ -1.1151 & 0.1055 & 0.0039 & 0.0035 \\ 0.0062 & -0.0682 & 0.001 & -0.0035 \\ -0.017 & 0.0049 & 0.1067 & 0.1692 \\ -0.0129 & 0.0106 & 0.2227 & 0.143 \\ 0.139 & 0.0059 & 0.0326 & -0.407 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} C \\ 0_{4\times 8} \end{bmatrix}, \qquad D_{11} = 0_{10\times 1}, \qquad D_{12} = \begin{bmatrix} 0_{6\times 4} \\ I_4 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 \\ 0.1 \\ 0 \\ 0 \\ 0.05 \\ 0 \end{bmatrix}$$

(HE4). S. Skogestad and I. Postlethwaite present in [56], section 12.2.2, the following model of a twin–engine, multi–purpose military helicopter. This eighth–order system represents the starting point of their study. The authors in [39] used this model with four inputs and six outputs.

$$A=\begin{bmatrix} 0 & 0 & 0 & 0.9986 & 0.0534 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.0032 & 0.0595 & 0 & 0 & 0 \\ 0 & 0 & -11.5705 & -2.5446 & -0.0636 & 0.1068 & -0.0949 & 0.0071 \\ 0 & 0 & 0.4394 & -1.9982 & 0 & 0.0167 & 0.0185 & -0.0012 \\ 0 & 0 & -2.0409 & -0.459 & -0.735 & 0.0193 & -0.0046 & 0.0021 \\ -32.1036 & 0 & -0.5034 & 2.2979 & 0 & -0.0212 & -0.0212 & 0.0158 \\ 0.1022 & 32.0578 & -2.3472 & -0.5036 & 0.8349 & 0.0212 & -0.0379 & 0.0004 \\ -1.911 & 1.7138 & -0.004 & -0.0574 & 0 & 0.014 & -0.0009 & -0.2905 \end{bmatrix},$$

$$B_1=I_8, \qquad B=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.1243 & 0.0828 & -2.7525 & -0.0179 \\ -0.0364 & 0.4751 & 0.0143 & 0 \\ 0.3045 & 0.015 & -0.4965 & -0.2067 \\ 0.2877 & -0.5445 & -0.0164 & 0 \\ -0.0191 & 0.0164 & -0.5445 & 0.2348 \\ -4.8206 & -0.0004 & 0 & 0 \end{bmatrix},$$

$$C_1=\begin{bmatrix} I_8 \\ 0_{4\times 8} \end{bmatrix}, \qquad D_{11}=0_{12\times 8}, \qquad D_{12}=\begin{bmatrix} 0_{8\times 4} \\ I_4 \end{bmatrix},$$

$$C=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.0595 & 0.0533 & -0.9968 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0535 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{21}=0_{6\times 8}$$

(HE5). A variation of the system above with eight state, two measurement and four control variables. The matrices A and B are the same as in *(HE4)*.

$$B_1^T=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

$$C_1=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.0595 & 0.0533 & -0.9968 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0535 & 1 & 0 & 0 & 0 \end{bmatrix}, \; D_{11}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; D_{12}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$C=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{21}=\begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \end{bmatrix}$$

(HE6). Starting from the helicopter model that was presented in *(HE4)* and *(HE5)* S. Skogestad and I. Postlethwaite developed this $\mathcal{H}_\infty$ mixed sensitivity design with four inputs, 20 states and six outputs. [56], section 12.2.3. The matrices A, B_1, B, C_1, D_{11}, D_{12}, C and D_{21} are given in file *"he6.mat"*.

(HE7). Like *(HE6)* with a difference in the matrices B_1, D_{11} and D_{21} (disturbance rejection design) [56], section 12.2.4. *"he7.mat"* contains all data matrices.

2.1.3. Jet engines.

(JE1). This system represents a multivariable servomechanism problem for a J–100 jet engine. It has three inputs, 30 states and five outputs [1], [15]. A, B and C are provided in file *"je1.mat"*. Additionally we set

$$B_1=I_{30}, \qquad C_1=\sqrt{0.5}\begin{bmatrix} C \\ 0_{3\times 30} \end{bmatrix}, \qquad D_{11}=0_{8\times 30}, \qquad D_{12}=\sqrt{0.5}\begin{bmatrix} 0_{5\times 3} \\ I_3 \end{bmatrix}, \qquad D_{21}=0_{5\times 30}$$

(JE2). We consider an aero–engine control problem that refers to a Rolls–Royce 2–spool reheated turbo fan, used to power modern military aircraft. The system has 21 states, three inputs and three outputs. For more information about this model see [56]. The data matrices A, B and C are given in file *"je2_3.mat"*

(JE3). S. Skogestad and I. Postlethwaite transformed the model above into a $\mathcal{H}_\infty$ design with 24 states, three inputs and six outputs [56]. The complete data set A, B_1, B, C_1, D_{11}, D_{12}, C and D_{21} is provided again in file *"je2_3.mat"*.

2.1.4. Reactor models.

(REA1). In the next two examples we consider models of a chemical reactor, both introduced in [34], p. 165. The data matrices are given by

$$A = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

(REA2). Obtained from *(REA1)* by leaving out the last row of the matrix C.

(REA3). A twelfth–order nuclear reactor model is given below [46]:

$$A = \begin{bmatrix} -0.4044 & 0 & 0 & 0.4044 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.4044 & 0 & 0 & 0.4044 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.4044 & 0 & 0 & 0.4044 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01818 & 0 & 0 & -0.5363 & 0 & 0 & 0.4045 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0818 & 0 & 0.4545 & -0.5363 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0818 & 0 & 0.4545 & -0.5363 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.15 & 0 & -0.15 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -7.5 & 0 & 0 & 75 & 0 & 0 & 600 & -74.995 & 0.033 & 0.346 & 0.621 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.475 & -0.033 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 25.95 & 0 & -0.346 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 46.57 & 0 & 0 & -0.621 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(REA4). Another model of a chemical reactor presented by P. M. Mäkilä [44]. The numerical values for this eight order one–input, one–output system are

$$A = \begin{bmatrix} 0.5623 & -0.01642 & 0.01287 & -0.0161 & 0.02094 & -0.02988 & 0.0183 & 0.00874 \\ 0.102 & 0.6114 & -0.02468 & 0.02468 & -0.03005 & 0.04195 & -0.02559 & 0.03889 \\ 0.1361 & 0.2523 & 0.641 & -0.03404 & 0.03292 & -0.04296 & 0.02588 & 0.08467 \\ 0.09951 & 0.2859 & 0.3476 & 0.6457 & -0.03249 & 0.03316 & -0.01913 & 0.1103 \\ -0.04794 & 0.08708 & 0.3297 & 0.3102 & 0.6201 & -0.03015 & 0.01547 & 0.08457 \\ -0.1373 & -0.1224 & 0.1705 & 0.3106 & 0.191 & 0.5815 & -0.01274 & 0.05394 \\ -0.1497 & -0.1692 & 0.1165 & 0.2962 & 0.1979 & 0.07631 & 0.5242 & 0.04702 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6065 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \; B = \begin{bmatrix} -0.1774 \\ -0.2156 \\ -0.2194 \\ -0.09543 \\ 0.0579 \\ 0.09303 \\ 0.08962 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.0465 & -0.1135 & -0.1909 & -0.2619 & -0.2634 & -0.1422 & -0.0002 & 0.1856 \end{bmatrix}, \; D_{11} = \begin{bmatrix} 0 \end{bmatrix}, \; D_{12} = \begin{bmatrix} 0.1001 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.0049 & 0.0049 & -0.006 & 0.01 & 0.0263 & 0.3416 & 0.6759 & 0 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 1 \end{bmatrix}$$

2.1.5. Decentralized interconnected systems.

(DIS1). A modification of the decentralized interconnected system of order eight presented by H. Singh, R. H. Brown and D. S. Naidu [54].

$$A = \begin{bmatrix} 0.144 & -0.058 & 0.056 & 0.042 & 0.12 & 2.1454 & 0 & 0.08 \\ -0.506 & -0.236 & -0.02 & -0.012 & -0.06 & -0.909 & 1.093 & -0.04 \\ 0 & 0 & -0.278 & 0.291 & 0 & 0 & 0 & 0.58 \\ 0 & 0 & 0 & -0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.303 & 0.029 & -1.67 & 0 & 0 & 0.092 \\ -0.154 & 0.133 & -0.006 & -0.004 & -0.014 & -1.688 & 0.236 & 0.013 \\ -0.345 & 0.304 & -0.018 & -0.014 & -0.032 & -0.611 & -1.824 & -0.024 \\ 0 & 0 & 0 & 0.247 & 0 & 0 & 0 & -1.978 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, B = \begin{bmatrix} -0.076 & 0.02 & 0 & 0 \\ 0.588 & -0.006 & 0 & 0 \\ 0 & 0.152 & 0 & 0 \\ 0 & 1.45 & 0 & 0 \\ 0 & 0 & 0 & 0.012 \\ 0 & 0 & 0.162 & -0.002 \\ 0 & 0 & 0.414 & -0.008 \\ 0 & 0 & 0 & 0.248 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} C \\ 0_{4\times 8} \end{bmatrix}, \qquad D_{11} = 0_{8\times 1}, \qquad D_{12} = \begin{bmatrix} 0_{4\times 4} \\ I_4 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{21} = 0_{4\times 1}$$

(DIS2). The following presents a decentralized control system with 2 control stations as introduced in [43].

$$A = \begin{bmatrix} -4 & 2 & 1 \\ 3 & -2 & 5 \\ -7 & 0 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(DIS3). A decentralized interconnected system provided by M. Saif and Y. Guan in the article [51].

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -8 & 1 & -1 & -1 & -2 & 0 \\ 4 & -0.5 & 0.5 & 0 & 0 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(DIS4). This sixth–order system consists of three subsystems [59].

$$A = \begin{bmatrix} 0 & 1 & 0.5 & 1 & 0.6 & 0 \\ -2 & -3 & 1 & 0 & 0 & 1 \\ 0 & 2 & 0.5 & 1 & 1 & 0.5 \\ 1 & 3 & 0 & 0.5 & 0 & -0.5 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ -3 & -4 & 0 & 0.5 & 0.5 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \quad C = I_6$$

(DIS5). Another decentralized control system introduced by M. C. de Oliveira, J. F. Camino and R. E. Skelton in [16].

$$A = \begin{bmatrix} 0.8189 & 0.0863 & 0.09 & 0.0813 \\ 0.2524 & 1.0033 & 0.0313 & 0.2004 \\ -0.0545 & 0.0102 & 0.7901 & -0.258 \\ -0.1918 & -0.1034 & 0.1602 & 0.8604 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0953 & 0 & 0 \\ 0.0145 & 0 & 0 \\ 0.0862 & 0 & 0 \\ -0.0011 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0045 & 0.0044 \\ 0.1001 & 0.01 \\ 0.0003 & -0.0136 \\ -0.0051 & 0.0936 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.1.6. Other examples.

(TG1). This system is a two–input, ten–state, two–output state–space description of the dynamics of a 1072 MVA nuclear powered turbo–generator. It can be found in [34], p. 117/167.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.11323 & -0.98109 & -11.847 & -11.847 & -63.08 & -34.339 & -34.339 & -27.645 & 0 \\ 324.121 & -1.1755 & -29.101 & 0.12722 & 2.83448 & -967.73 & -678.14 & -678.14 & 0 & -129.29 \\ -127.3 & 0.46167 & 11.4294 & -1.0379 & 13.1237 & 380.079 & 266.341 & 266.341 & 0 & 1054.85 \\ -186.05 & 0.67475 & 16.7045 & 0.86092 & -17.068 & 555.502 & 389.268 & 389.268 & 0 & -874.92 \\ 341.917 & 1.09173 & 1052.75 & 756.465 & 756.465 & -29.774 & 0.16507 & 3.27626 & 0 & 0 \\ -30.748 & -0.09817 & -94.674 & -68.029 & -68.029 & 2.67753 & -2.6558 & 4.88497 & 0 & 0 \\ -302.36 & -0.96543 & -930.96 & -668.95 & -668.95 & 26.3292 & 2.42028 & -9.5603 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.6667 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -10 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1.6667 & 0 \\ 0 & 10 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.49134 & 0 & -0.63203 & 0 & 0 & -0.20743 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(AGS). Another example by Y. S. Hung and A. G. J. MacFarlane [34], p. 27/163. The system is a two–input, twelve–state, two–output model of an automobile gas turbine.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.202 & -1.15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.36 & -13.6 & -12.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.62 & -9.4 & -9.15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -188 & -111.6 & -116.4 & -20.8 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1.0439 & 4.1486 \\ 0 & 0 \\ 0 & 0 \\ -1.794 & 2.6775 \\ 0 & 0 \\ 0 & 0 \\ 1.0439 & 4.1486 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1.794 & 2.6775 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.264 & 0.806 & -1.42 & -15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.9 & 2.12 & 1.95 & 9.35 & 25.8 & 7.14 & 0 \end{bmatrix}$$

(WEC1). M. Steinbuch presents the following models for a wind energy conversion system at different wind speed [57], Appendix A. The design goal is to minimize fluctuations in speed and current of a wind turbine while reducing mechanical fatigue load. This first model refers to a wind speed of 12 m/s.

$$A = \begin{bmatrix} -5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5.5005 & -1479.1 & -3.2812 & -0.017889 & 0 & 0 & 169.68 & 36.137 & 36.137 & 144.83 \\ 0 & 1416.4 & 3.125 & 0 & 0 & 0 & -169.68 & -36.137 & -36.137 & -144.83 \\ 0 & 0 & 0 & 0.095493 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.8416 & 0 & 0.11552 & -1257.1 & 1015.1 & 1011.1 & 499.09 \\ 0 & 0 & 0 & 4.6042 & 0 & 2.096 & -693.13 & 559.33 & 631.31 & 306.18 \\ 0 & 0 & 0 & 5.7968 & 0 & -1.8671 & -976.81 & 788.51 & 708.25 & 355.08 \\ 0 & 0 & 0 & -2.8663 & 0 & -0.047856 & 413.58 & -343.35 & -341.63 & -212.45 \end{bmatrix}, B = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & -305.65 \\ 0 & 0 & -166.27 \\ 0 & 0 & -239.88 \\ 0 & 0 & 96.02 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.045455 & 0.045455 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12.249 & 0.027025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(WEC2). The wind energy conversion system at a wind speed of 16 m/s.

$$A(3,1:4) = \begin{bmatrix} -33.69 & -1479.1 & -3.3531 & -0.089802 \end{bmatrix},$$

Rest of A, B and C like in *(WEC1).*

(WEC3). The wind energy conversion system at a wind speed of 20 m/s.

$$A(3,1:4) = \begin{bmatrix} -70.878 & -1479.1 & -3.4321 & -0.16877 \end{bmatrix},$$

Rest of A, B and C like in *(WEC1).*

(HF1). A very simple one–dimensional example which describes the control of the heat flow in a thin rod [32], ex. 4.1. The input u of the system acts at one of the ends of the rod. The chosen order in this case is $n_x = 130$.

$$A = \frac{1}{h}\begin{bmatrix} -1 & 1 & & \\ 1 & -2 & \ddots & \\ & \ddots & \ddots & 1 \\ & & 1 & -2 \end{bmatrix} \in \Re^{130\times 130}, \qquad B_1 = B, \qquad B = \begin{bmatrix} 0_{129\times 1} \\ \frac{1}{h} \end{bmatrix}, \quad h = \frac{1}{n_x+1},$$

$$C_1 = C, \qquad D_{11} = 0_{2\times 1}, \qquad D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} & 0_{2\times 63} & \begin{bmatrix} 0 \\ 1 \end{bmatrix} & 0_{2\times 65} \end{bmatrix}, \qquad D_{21} = 0_{2\times 1}$$

(BDT1). This model represents a fairly realistic model of a binary distillation tower with pressure variation included in the model's description. The system is multivariable, with three control inputs and three outputs, and includes one disturbance input. It was taken from E. J. Davison [14].

$$A = \begin{bmatrix}
-0.014 & 0.0043 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0095 & -0.0138 & 0.0046 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0005 \\
0 & 0.0095 & -0.0141 & 0.0063 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0002 \\
0 & 0 & 0.0095 & -0.0158 & 0.011 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0095 & -0.0312 & 0.015 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0202 & -0.0352 & 0.022 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0202 & -0.0422 & 0.028 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.0202 & -0.0482 & 0.037 & 0 & 0.0002 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0202 & -0.0572 & 0.042 & 0.0005 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0202 & -0.0483 & 0.0005 \\
0.0255 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0255 & -0.0185
\end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.01 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B = \begin{bmatrix}
0 & 0 & 0 \\
0.000005 & -0.00004 & 0.0025 \\
0.000002 & -0.00002 & 0.005 \\
0.000001 & -0.00001 & 0.005 \\
0 & 0 & 0.005 \\
0 & 0 & 0.005 \\
-0.000005 & 0.00001 & 0.005 \\
-0.00001 & 0.00003 & 0.005 \\
-0.00004 & 0.000005 & 0.0025 \\
-0.00002 & 0.000002 & 0.0025 \\
0.00046 & 0.00046 & 0
\end{bmatrix},$$

$$C_1 = \begin{bmatrix} C \\ 0_{3\times 11} \end{bmatrix}, \qquad D_{11} = 0_{6\times 1}, \qquad D_{12} = \begin{bmatrix} 0_{3\times 3} \\ I_3 \end{bmatrix},$$

$$C = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, \qquad D_{21} = 0_{3\times 1}$$

(BDT2). Another model of a binary distillation tower by S. Skogestad and I. Postlethwaite. It has four inputs, four outputs, two disturbances and 82 states. A detailed description of this model can be found in [56], section 12.4. The matrices A, B_1, B, C_1, D_{12} and C are given in file *"bdt2.mat"*, while D_{11} and D_{21} are set to be 0.

(MFP). This system is a moored floating platform. The platform is anchored to the bottom of the ocean and equipped with two thrusters. The goal is to minimize the drift resulting from wave action by appropriate master control. It is described by C. Scherer, P. Gahinet and M. Chilali in [52].

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.101 & -0.1681 & -0.04564 & -0.01075 \\ 0.06082 & -2.1407 & -0.05578 & -0.1273 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.1179 & 0.1441 & 0.1476 \\ 0.1441 & 1.7057 & -0.7557 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(UWV). We consider in this problem a control surface servo for an underwater vehicle. A detailed description of this model can be found in [14].

$$A = \begin{bmatrix} 0 & 850 & 0 & 0 & 0 & 0 & 0 & 0 \\ -850 & -120 & -4100 & 0 & 0 & 0 & 0 & 0 \\ 33 & 0 & -33 & 0 & -700 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1400 & 0 & 0 & 0 \\ 0 & 0 & 1600 & -450 & -110 & 0 & 0 & 0 \\ 0 & 0 & 0 & 81 & 0 & -1 & 0 & -900 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 110 \\ 0 & 0 & 0 & 0 & 0 & 12 & -1.1 & -22 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 & 0 \\ 9900 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 99 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 4.6 & 99000 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(IH). The data presented here describes a mathematical model of position and velocity control for a string of high–speed vehicles. The problem is also known as "smart highway" or "intelligent highway" and was borrowed from J. Abels and P. Benner [1]. The here chosen version has eleven inputs, 21 states and ten outputs. The matrices A, B and C are given in file *"ih.mat"*. Furthermore we defined

$$B_1 = I_{21}, \qquad C_1 = \sqrt{5} \begin{bmatrix} C \\ 0_{1\times 21} \end{bmatrix}, \qquad D_{11} = 0_{10\times 21}, \qquad D_{12} = \sqrt{0.5}\, I_{11}, \qquad D_{21} = 0_{10\times 21}.$$

(CSE1). This is a model of a string consisting of coupled springs, dashpots and masses. The inputs are two forces, one acting on the left end of the spring, the other on the right end [1], [31]. Here we set the order equal to 20 ($l = 10$).

$$l = 10, \qquad \mu = 4, \qquad \delta = 4, \qquad \kappa = 1,$$
$$M = \mu I_l, \qquad L = \delta I_l, \qquad N = I_l, \qquad P = I_l$$

$$K = \kappa \begin{bmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & -1 \end{bmatrix},$$

$$A = \begin{bmatrix} 0_{l\times l} & I_l \\ -M^{-1}K & -M^{-1}L \end{bmatrix}, \qquad B_1 = -0.02\, B(:,2), \qquad B = \begin{bmatrix} 0_{l\times 2} \\ -M^{-1}D \end{bmatrix},$$

$$C_1 = \begin{bmatrix} C \\ 0_{2\times 2l} \end{bmatrix}, \qquad D_{11} = 0_{(l+2)\times 1}, \qquad D_{12} = \begin{bmatrix} 0_{l\times 2} \\ I_2 \end{bmatrix},$$

$$C = \begin{bmatrix} N & P \end{bmatrix}, \qquad D_{21} = 0_{l\times 1}$$

(CSE2). Like above with $l = 30$ and therefore a resulting order of 60.

(EB1). This example consists of a simple supported Euler–Bernoulli beam. It was adopted from [27]. The damping coefficient was set to $\xi = 10^{-2}$ (low damping).

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -0.02 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -16 & -0.08 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -81 & -0.18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -256 & -0.32 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -625 & -0.5 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 & 0 \\ 0.9877 & 0 \\ 0 & 0 \\ -0.309 & 0 \\ 0 & 0 \\ -0.891 & 0 \\ 0 & 0 \\ 0.5878 & 0 \\ 0 & 0 \\ 0.7071 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0.9877 \\ 0 \\ -0.309 \\ 0 \\ -0.891 \\ 0 \\ 0.5878 \\ 0 \\ 0.7071 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0.809 & 0 & -0.9511 & 0 & 0.309 & 0 & 0.5878 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \\ 1.9 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0.9877 & 0 & -0.309 & 0 & -0.891 & 0 & 0.5878 & 0 & 0.7071 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 & 1.9 \end{bmatrix}$$

(EB2). Like *(EB1)* with different performance criteria yielding a change in the following matrices:

$$C_1 = \begin{bmatrix} 0.809 & 0 & -0.9511 & 0 & 0.309 & 0 & 0.5878 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},$$

(EB3). Like *(EB2)* with a damping coefficient set to $\xi = 10^{-7}$ (very low damping). This causes a change in matrix A:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -2\cdot 10^{-7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -16 & -8\cdot 10^{-7} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -81 & -18\cdot 10^{-7} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -256 & -32\cdot 10^{-7} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -625 & -50\cdot 10^{-7} \end{bmatrix}$$

(EB4). Like *(EB2)* with a damping coefficient set to $\xi = 10^{-7}$ (very low damping) and an increased order from ten to 20.

(EB5). Like *(EB2)* with a damping coefficient set to $\xi = 10^{-7}$ (very low damping) and an increased order of 40.

(EB6). Like *(EB2)* with a damping coefficient set to $\xi = 10^{-7}$ (very low damping) and an increased order of 160.

(PAS). This model describes a piezoelectric bimorph actuator system design. For detailed information about this case study see [11], p. 283. The data matrices are given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -274921.63 & -73.2915 & -274921.63 & 0 & 0 \\ 0 & 0 & -0.9597 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ -274921.63 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.12841 \\ -3.39561\cdot 10^{-7} \\ 0 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(TF1). E. Gershon, U. Shaked and I. Yaesh present this terrain following model in [28].

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ -0.088 & 0.0345 & 0 & 0 & 1 & -0.0032 & 0 \\ 0 & 0 & 0.05 & 0 & 0 & 0 & -0.00001 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.05 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.09 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2.23 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \sqrt{3} & 0 \\ 0 & \sqrt{0.3} \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0.04 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(TF2). Like *(TF1)* with a different sensor matrix C.

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0.04 \\ 0 \\ 0 \end{bmatrix}$$

(TF3). Another sensor matrix C for the terrain following model.

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0.04 \\ 0 \\ 0 \end{bmatrix}$$

(PSM). A model of a two–area interconnected power system [60], [22].

$$A = \begin{bmatrix} -0.04165 & 0 & 4.92 & -4.92 & 0 & 0 & 0 \\ -5.21 & -12.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.33 & -3.33 & 0 & 0 & 0 & 0 \\ 0.545 & 0 & 0 & 0 & -0.545 & 0 & 0 \\ 0 & 0 & 0 & 4.92 & -0.04165 & 0 & 4.92 \\ 0 & 0 & 0 & 0 & -5.21 & -12.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.33 & -3.33 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 & 0 \\ 12.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 12.5 \\ 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} -4.92 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -4.92 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} C \\ 0_{2\times 7} \end{bmatrix}, \qquad D_{11} = 0_{5\times 2}, \qquad D_{12} = \begin{bmatrix} 0_{3\times 2} \\ I_2 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \qquad D_{21} = 0_{3\times 2}$$

(TL). This example describes a transmission line which is a circuit model of the impendence of interconnect structures accounting for both the charge accumulation on the surface of conductors and the current travelling along conductors [10], [42], [45]. The system has two inputs, 256 states and two outputs. The data matrices A, B, C and E are provided in file *"tl.mat"*. Note, the linear model is of descriptor type with regular E, i. e. $E\dot{x} = Ax + Bu$, $y = Cx$. The matrices provided by our MATLAB–file are therefore already set to:

$$A := E^{-1}A, \qquad B_1 := E^{-1}I, \qquad B := E^{-1}B$$

(CDP). A 120th–order model of a compact disk player with two inputs and two outputs. The control task is to achieve track following, which basically amounts to pointing the laser spot to the track of pits on the CD that is rotating. The challenge is to find a low–cost controller that can make the servo–system faster and less sensitive to external shocks [10], [17]. The matrices A, B and C are given in file *"cdp.mat"*. The other system matrices are defined as follows.

$$B_1 = 0.01\,B, \quad C_1 = \begin{bmatrix} C \\ 0_{2\times 120} \end{bmatrix}, \quad D_{11} = 0_{4\times 2}, \quad D_{12} = \sqrt{2}\begin{bmatrix} 0_{2\times 2} \\ I_2 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & 1 \\ 0.03 & 0.00002 \end{bmatrix}$$

2.1.7. Academic test problems .

(NN1). This first simple example was presented by L. F. Miller *et al.* [46]. The system matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 13 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 5 & -1 \\ -1 & -1 & 0 \end{bmatrix}$$

(NN2). A classical example in the output feedback literature [41] is given by

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

(NN3). C. W. Scherer introduced the following system [53]:

$$A = \begin{bmatrix} 0.5 & 1 & 1.5 & 1 \\ -1 & 3 & 2.1 & 2 \\ 1 & -1 & -0.6 & 1 \\ -2 & 2 & -1 & 1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 \end{bmatrix}$$

(NN4). A fourth–order example from [46] is given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -2.93 & -4.75 & -0.78 \\ 0.086 & 0 & -0.11 & -1 \\ 0 & -0.042 & 2.59 & -0.39 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & -3.91 \\ 0.035 & 0 \\ -2.53 & 0.31 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(NN5). The following system represents a seventh–order, single–input, two–output model of a Saturn V booster, which was presented in [46].

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & -0.65 & -0.002 & 2.6 & 0 \\ -0.014 & 1 & -0.041 & 0.0002 & -0.015 & -0.033 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -45 & -0.13 & 255 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -50 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(NN6). H. P. Horisberger and P. R. Bélanger introduced the following ninth–order system with one input and four outputs [33].

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -20 & -4.2 & 0 & 4.45 & 12.5 & 0 & 100 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.7 & 8.35 & 0 & -1.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 10.9 & 0 & 0 & -2.55 & -250 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 5.9 & 0 & 0 & -1.39 & 0 & 0 & -3700 & 0 \end{bmatrix}, \quad B_1 = \sqrt{0.1}\, I_9, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3.3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_1 = \sqrt{10}\, I_9, \qquad D_{11} = 0_{9\times 9}, \qquad D_{12} = \begin{bmatrix} 0_{8\times 1} \\ 10 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0.66 & 0 & 1.2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0.66 & 0 & 1.2 \end{bmatrix}, \qquad D_{21} = 0_{4\times 9}$$

(NN7). A modification of *(NN6)* (changes in B_1, C_1, D_{11}, D_{12} and D_{21}).

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -20 & -4.2 & 0 & 4.45 & 12.5 & 0 & 100 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.7 & 8.35 & 0 & -1.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 10.9 & 0 & 0 & -2.55 & -250 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 5.9 & 0 & 0 & -1.39 & 0 & 0 & -3700 & 0 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0.145 & 0.478 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0.0523 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.598 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3.3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D_{11} = 0_{3\times 5}, \qquad D_{12} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0.66 & 0 & 1.2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0.66 & 0 & 1.2 \end{bmatrix}, \qquad D_{21} = 0_{4\times 5}$$

(NN8).

$$A = \begin{bmatrix} -0.2 & 0.1 & 1 \\ -0.05 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 \\ 0 & 0.7 \\ 1 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(NN9). This system was introduced by B. M. Chen [11] p. 110/119.

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 5 & 1 \\ 0 & 0 \\ 0 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & -2 & -3 & -2 & -1 \\ 1 & 2 & 3 & 2 & 1 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

(NN10). A three–input, three–output system of order eight taken from [61].

$$A = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -7 & 0 & 0 & -2 \\ 0 & -1 & 0 & 1 & 4 & 0 & 0 & 2 \\ 0 & -2 & 0 & 0 & 2 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & -2 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}, \quad B_1 = 0_{8\times 3}, \quad B = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ -1 & -1 & -3 \\ 1 & 0 & 1 \\ 0 & 2 & 4 \\ 2 & 1 & 5 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_{11} = 0_{2\times 3}, \quad D_{12} = 0_{2\times 3},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad D_{21} = 0_{3\times 3}$$

(NN11). A larger problem of order sixteen given by P. Appkarian and H. D. Tuan [4].

$$A = \left[\begin{array}{cccccccccccccccc} -101 & -99.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -101 & -99.9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 427.098 & -46.8341 & -1 & 0 & 0.4271 & -0.0468 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 232.0719 & 120.4649 & 0 & -1 & 0.2321 & 0.1205 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -764.2456 & 85.4154 & 0 & 0 & -1.7642 & 0.0854 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 166.827 & -264.7739 & 0 & 0 & 0.1668 & -1.2648 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.3162 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.1 & -0.0759 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3162 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.1 & -0.0759 \\ 0 & 0 & 0 & 0 & 0 & -0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{array}\right],$$

$$B_1 = \begin{bmatrix} 0 & -0.001 & 0 \\ 0 & -0.001 & 0 \\ 0 & 0 & -0.001 \\ 0 & 0 & -0.001 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.1787 & 0.0003 & 0.0001 \\ -0.8364 & 0.0001 & 0.0003 \\ 0.0818 & -0.0005 & -0.0003 \\ 0.3577 & 0.0001 & -0.0003 \\ 0 & -0.3162 & 0 \\ 0 & 0.125 & 0 \\ -0.3162 & 0 & 0 \\ 0 & 0 & 0.125 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -9.995 & 0 \\ 0.199 & -9.995 & 0 \\ 0.211 & 0 & -9.995 \\ -0.233 & 0 & -9.995 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 2.7173 & 1.4274 \\ 0 & 1.4274 & 2.8382 \\ 0 & -4.7909 & -2.6032 \\ 0 & 1.0261 & -2.6393 \\ 0.11 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.01 & 0 & 0 \end{bmatrix},$$

$$C_1 = \left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 1.5564 & 3.4834 & 0 & 0 & 0.0016 & 0.0035 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4743 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3479 & 0 \end{array}\right], \; D_{11} = 0_{3\times 3}, \; D_{12} = 0_{3\times 3},$$

$$C = \left[\begin{array}{cccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3162 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3162 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.5564 & 3.4834 & 0 & 0 & 0.0016 & 0.0035 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4743 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.3479 & 0 \end{array}\right], \; D_{21} = 0_{5\times 3}$$

(NN12). A two–input, two–output system which can be found in [50], example 3.21.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & -3 \\ 0 & 0 \\ 0 & 1 \\ 0 & -1 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(NN13). This sixth–order system was presented by D.–W. Gu, P. Hr. Petkov and M. M. Konstantinov [29], p. 15.

$$A = \begin{bmatrix} -1 & 0 & 4 & 5 & -3 & -2 \\ -2 & 4 & -7 & -2 & 0 & 3 \\ -6 & 9 & -5 & 0 & 2 & -1 \\ -8 & 4 & 7 & -1 & -3 & 0 \\ 2 & 5 & 8 & -9 & 1 & -4 \\ 3 & -5 & 8 & 0 & 2 & -6 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -3 & -4 & -2 \\ 2 & 0 & 1 \\ -5 & -7 & 0 \\ 4 & -6 & 1 \\ -3 & 9 & -8 \\ 1 & -2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ -5 & 2 \\ 7 & -2 \\ 1 & -2 \\ 0 & 5 \\ -6 & -2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & -1 & 2 & -4 & 0 & -3 \\ -3 & 0 & 5 & -1 & 1 & 1 \\ -7 & 5 & 0 & -8 & 2 & -2 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 4 & 0 \\ 5 & -3 & -4 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 9 & -3 & 4 & 0 & 3 & 7 \\ 0 & 1 & -2 & 1 & -6 & -2 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(NN14). The same as *(NN13)* with these changes:

$$D_{11} = 0_{3\times 3}, \qquad D_{12} = \begin{bmatrix} -4 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

(NN15). A space backpack model that has been used in [49]. It is described by the following state space matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -79.285 & -0.113 & 0 \\ 28.564 & 0.041 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0.041 \\ -0.03 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0.041 & -0.0047 \\ -0.03 & -0.0016 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(NN16). The next example illustrates an application for a large space structure. The model was taken from [9].

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.42 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1849 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4.41 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4.84 & 0 \end{bmatrix}, \quad B_1 = I_8, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.92 & -1.4 & 0.92 & -1.4 \\ 0 & 0 & 0 & 0 \\ 0.65 & 1.6 & 0.65 & -1.6 \\ 0 & 0 & 0 & 0 \\ 1.4 & -1 & 1.4 & 1 \\ 0 & 0 & 0 & 0 \\ 2 & -0.8 & -2 & -0.8 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.065 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.065 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0_{2\times 8} & & & & \end{bmatrix}, \qquad D_{11} = 0_{4\times 8}, \qquad D_{12} = I_4,$$

$$C = \begin{bmatrix} 0 & -1.8 & 0 & 1.3 & 0 & 2.9 & 0 & 4.1 \\ 0 & -2.7 & 0 & 3.2 & 0 & -2.1 & 0 & -1.6 \\ 0 & 1.8 & 0 & 1.3 & 0 & 2.9 & 0 & -4.1 \\ 0 & -2.7 & 0 & -3.2 & 0 & 2.1 & 0 & -1.6 \end{bmatrix}, \qquad D_{21} = 0_{4\times 8}$$

(NN17). A simple example by P. Gahinet and A. J. Laub with a rank–deficient matrix D_{12} [24].

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 \end{bmatrix}$$

(NN18). This example is a dynamical system of order 1006 with one input and one output. [10], [48]. Note that matrix A is provided in a sparse format.

$$A = \begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & A_3 & \\ & & & A_4 \end{bmatrix}, \; A_1 = \begin{bmatrix} -1 & 100 \\ -100 & -1 \end{bmatrix}, \; A_2 = \begin{bmatrix} -1 & 200 \\ -200 & -1 \end{bmatrix}, \; A_3 = \begin{bmatrix} -1 & 400 \\ -400 & -1 \end{bmatrix},$$

$$A_4 = -diag\,(1, 2, ..., 1000), \qquad B^T = C = [\, \underbrace{10 \,...\, 10}_{6} \; \underbrace{1 \,...\, 1}_{1000} \,], \qquad B_1 = 0.01 \cdot B$$

$$C_1 = \begin{bmatrix} C \\ 0_{1\times 1006} \end{bmatrix}, \quad D_{11} = 0_{2\times 1}, \quad D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad D_{21} = 0$$

2.2. 2D heat flow examples [37] .

(HF2D1) – (HF2D18). In this paragraph we state a quick discussion of the (original) infinite dimensional 2D heat flow models which are currently implemented in *COMPl$_e$ib* . Using standard discretization schemes we obtain large scale finite dimensional approximations to the infinite dimensional control problems (i, e. see [37, Section 3]). In *COMPl$_e$ib* we state the data matrices of the corresponding discretized control problems. In particular, the discretization of the two dimensional heat flow models stated in [37, Section3], [40] yields (in general) a large scale nonlinear and perturbed control system of the following form:

$$\begin{aligned} E\dot{x}(t) &= (A + \delta A)x(t) + G(x(t)) + B_1 w(t) + Bu(t), \quad x(0) = x_0, \\ z(t) &= C_1 x(t) + D_{12} u(t), \\ y(t) &= Cx(t), \end{aligned} \tag{2.1}$$

where $E \in \boldsymbol{R}^{n_x \times n_x}$ is a regular diagonal matrix and the matrices C_1 and D_{12} are defined by $C_1 = \sqrt{0.5c_1}\,[I_{n_x} \;\; 0_{n_u \times n_x}]^T$, $D_{12} = \sqrt{0.5d_1}\,[0_{n_x \times n_u} \;\; I_{n_u}]^T$ with given positive scalars $c_1, d_1 \in \boldsymbol{R}$. If $\delta A \equiv 0$ the system matrix A is not affected by a perturbation, and, if $G(x(t)) \equiv 0$, the system is linear. Depending on the corresponding heat flow model, one get linear or nonlinear control systems which can be controlled by a static output feedback control law of the form $u(t) = Fy(t)$, where $F \in \boldsymbol{R}^{n_u \times n_y}$ denotes an unknown SOF gain. For more details, we refer the interested reader to the case studies of these models in [37, Section3] and [40].

The first nine examples represent the approximation of the discretized 2D heat flow models, while the other nine are the corresponding highly reduced order approximations of the large dimensional systems gained by the proper orthogonal decomposition (POD) approach as discussed in [40].

Due to the regularity of the so–called descriptor matrix E, it is possible to reduce the more general control system (2.1) to (1.1). In particular, if applicable, neglecting the nonlinear term $G(x(t)$ in (2.1), and redefining the data matrices $A + \delta A$, B_1, B by

$$A := E^{-1}(A + \delta A), \quad B_1 := E^{-1}B_1, \quad B := E^{-1}B$$

yields the equivalent standard linear system format (1.1) of *COMPl$_e$ib* . Thus, in *COMPl$_e$ib* , the corresponding MATLAB function returns the redefined data matrices of the (equivalent linearized) system if we call one of these heat flow models. The last column of Table 2.2 refers to the property of the data matrix A (stable/unstable) and states the source of the original model (linear/nonlinear). Finally, note, we have subdivided this group into following two problem sets: the first set contains all large scale and typically sparse models (HF2D1 – HF2D9) while the second problem set collects the low dimensional POD approximations (HF2D10 – HF2D18) of the large dimensional models.

2.3. Second order models. This example class of *COMPl$_e$ib* represents so–called second order models of the form:

$$M\ddot{q} + D\dot{q} + Sq = \hat{B}u, \quad M \text{ mass, } D \text{ damping, } S \text{ stiffness matrix} \tag{2.2}$$

which can be transformed into the first order standard system (1.1) by defining:

$$x := \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad A := \begin{bmatrix} 0 & I \\ -M^{-1}D & -M^{-1}S \end{bmatrix}, \quad B := \begin{bmatrix} 0 \\ M^{-1}\hat{B} \end{bmatrix}. \tag{2.3}$$

Note, in this case, the system matrices have a special structure. This is the reason why we have collected those problems in an extra class. But, note, all currently *COMPl$_e$ib* examples in this problem class are also SOF stabilizable which, in general, is not always true for second order models. Finally, this *COMPl$_e$ib* problem class consists of the following problem sets:

- six so–called cable mass models with very low damping *(CM)*
- three models of a space structure developed by the "Deutsche Forschungsanstalt für Luft- und Raumfahrt" *(DLR)*
- two instances of a component of the International Space Station *(ISS)*
- some other second order models, i. e. a tuned mass damper *(TMD)* example, a clamped beam model *(CBM)*, a flexible satellite *(FS)* example, and, finally, a model of the Los Angeles (university) hospital *(LAH)*

(CM1). The cable mass model is borrowed from [7] and [47]. It describes a hybrid distributed parameter system and represents the non–linear dynamic response of a relief valve used to protect a pneumatic system from overpressure. The damping coefficient has been chosen to be $5 \cdot 10^{-5}$ (low damping). It is a one–input, two–output model, with an order depending on the subdivision of a distance interval. Here we present an 20th–order model. The data–matrices A, B_1, B, C_1, D_{11}, D_{12}, C and D_{21} are provided in file *"cm1.mat"* and are of the form:

$$A = \begin{bmatrix} 0_{10\times 10} & I_{10} \\ \multicolumn{2}{c}{\tilde{A}} \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0_{10\times 1} \\ \tilde{B}_1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0_{10\times 1} \\ \tilde{B} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0_{3\times 9} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & 0_{3\times 9} & \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{bmatrix}, \qquad D_{11} = 0_{3\times 1}, \qquad D_{12} \approx \begin{bmatrix} 0 \\ 0 \\ 1.2247 \end{bmatrix},$$

$$C = \begin{bmatrix} 0_{2\times 9} & \begin{bmatrix} 1 \\ 0 \end{bmatrix} & 0_{2\times 9} & \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(CM2). The to *(CM1)* corresponding example of order 60. Data given in file *"cm2.mat"*.
(CM3). The to *(CM1)* corresponding example of order 120. Data given in file *"cm3.mat"*.

(CM4). The to *(CM1)* corresponding example of order 240. Data given in file *"cm4.mat"*.
(CM5). The to *(CM1)* corresponding example of order 480. Data given in file *"cm5.mat"*.
(CM6). The to *(CM1)* corresponding example of order 960. Data given in file *"cm6.mat"*.
(TMD). We present here the design of a two–degree-of–freedom tuned–mass–damper to attenuate a motion of a single mode of a primary system, developed by L. Zuo and A. Nayfeh. For the choice of the parameters and a detailed description of this model see the provided MATLAB–file and [63].

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.6 & 0 & 0 & -0.01 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0.01 \\ 0 \\ 0 \\ 0.5999 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1.625 & 0.625 \\ 0.625 & -1.625 \\ 0.2 & 0.2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 \\ -0.01 \\ 0 \\ -0.01 \end{bmatrix}$$

(FS). A control system design for a flexible satellite presented by H. Buschek and A. J. Calise [8].

$$M = \begin{bmatrix} 77511 & 248.1 \\ 248.1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0.002288 \end{bmatrix}, \quad K = \begin{bmatrix} 0 & 0 \\ 0 & 0.098696 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$A = \begin{bmatrix} 0_{2\times 2} & & I_2 & 0_{2\times 1} \\ -M^{-1}K & & -M^{-1}D & 0_{2\times 1} \\ 1 & 0 & 0 \quad 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ M^{-1}\mathcal{B} \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(DLR1). This model developed by J. Bals and the "Deutsche Forschungsanstalt für Luft- und Raumfahrt" (DLR, Oberpfaffenhofen, Germany) describes the so–called "plate experiment" for the active vibration damping of large flexible space structures [5]. This already reduced system is of order ten.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -8.4268 & 0 & 9.6557 & 0 & -5.083 & -0.0253 & 0 & 0.0155 & 0 & -0.0112 \\ 0 & -20.2022 & 0 & 20.0736 & 0 & 0 & -0.0244 & 0 & 0.0151 & 0 \\ -23.9425 & 0 & -10.7354 & 0 & -147.0685 & -0.0049 & 0 & -0.0359 & 0 & -0.0849 \\ 0 & 126.1547 & 0 & -132.8028 & 0 & 0 & 0.0947 & 0 & -0.1089 & 0 \\ -39.905 & 0 & 6.607 & 0 & -188.4411 & -0.0247 & 0 & 0.0016 & 0 & -0.1368 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -0.0076 & -0.0076 \\ -0.0351 & 0.0351 \\ 0.0972 & 0.0972 \\ -0.1824 & 0.1824 \\ 0.0748 & 0.0748 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -0.001 & -0.001 \\ -0.0133 & 0.0133 \\ 0.048 & 0.048 \\ -0.0516 & 0.0516 \\ 0.0213 & 0.0213 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.8084 & 0 & 0.7509 & 0 & -0.9501 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.0115 & -0.0536 & 0.9713 & -0.2009 & -0.5746 \\ 0 & 0 & 0 & 0 & 0 & 0.0115 & 0.0536 & 0.9713 & 0.2009 & -0.5746 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & 0 \\ 0.0972 & 0.7509 \end{bmatrix}$$

(DLR2). The original system of order 40 corresponding to *(DLR1)*. The data matrices A, B_1, B, C_1, D_{11}, D_{12}, C and D_{21} are given in *"dlr2_3.mat"*.

(DLR3). Like *(DLR2)* with a change in the sensor structure, i.e. the matrix C. The complete data set is given in the same file as above.

(ISS1). A structural model of component 1R (the Russian service module) of the International Space Station (ISS) [10], [30]. It has 270 states, three inputs and three outputs. File *"iss1_2.mat"* contains the data matrices A, B and C. We added:

$$B_1 = 0.01\ B(:,1), \quad C_1 = \sqrt{100000} \begin{bmatrix} I_{270} \\ 0_{3\times 270} \end{bmatrix}, \quad D_{11} = 0_{273\times 1}, \quad D_{12} = \begin{bmatrix} 0_{270\times 3} \\ I_3 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 \\ 0 \\ 0.05 \end{bmatrix}$$

(ISS2). The first and the second half of row one in the sensor matrix C from *(ISS1)* have been exchanged, while all other matrices are like in the previous case.

(CBM). The clamped beam model has 348 states and is a single input, single output system. The input represents the force applied to a structure, and the output the resulting displacement [3], [10]. A, B and C are given in file *"cbm.mat"*. We added:

$$B_1 = 0.09\ B, \quad C_1 = \begin{bmatrix} C \\ 0_{1\times 348} \end{bmatrix}, \quad D_{11} = 0_{2\times 1}, \quad D_{12} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}, \quad D_{21} = 0.05$$

(LAH). An 48th order, one–input, one–output model of a building (the Los Angeles University Hospital) with eight floors each having three degrees of freedom, namely displacement in x and y directions and rotation [10], [3]. The matrices A, B and C are provided in file *"lah.mat"*. Additionally we defined:

$$B_1 = 0.01\ B, \quad C_1 = \begin{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & 0_{3\times 23} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & 0_{3\times 23} \end{bmatrix}, \quad D_{11} = 0_{3\times 1}, \quad D_{12} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad D_{21} = 0.05$$

2.4. Reduced order control examples . The last class of examples in *COMPl$_e$ib* 1.0 are so–called reduced order control problems. These instances are not SOF stabilizable, but they are at least stabilizable by a reduced order output feedback control law of order $n_c \ll n_x$. Using a well known system augmentation technique, the considered system has the following form

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{x}_c(t) \end{bmatrix} &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} 0 & B \\ I & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_c(t) \\ u(t) \end{bmatrix} \\ z(t) &= \begin{bmatrix} C_1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} + D_{11} w(t) + \begin{bmatrix} 0 & D_{12} \end{bmatrix} \begin{bmatrix} \dot{x}_c(t) \\ u(t) \end{bmatrix} \\ \begin{bmatrix} x_c(t) \\ y(t) \end{bmatrix} &= \begin{bmatrix} 0 & I \\ C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ D_{21} \end{bmatrix} w(t) \end{aligned} \tag{2.4}$$

and the reduced order (dynamic) output feedback control law

$$\begin{bmatrix} \dot{x}_c(t) \\ u \end{bmatrix} = F \begin{bmatrix} x_c(t) \\ y \end{bmatrix}, \quad F := \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \tag{2.5}$$

looks like a static output feedback controller, where $x_c \in \boldsymbol{R}^{n_c}$, $A_c \in \boldsymbol{R}^{n_c\times n_c}$, $B_c \in \boldsymbol{R}^{n_c\times n_y}$, $C_c \in \boldsymbol{R}^{n_u\times n_c}$, $D_c \in \boldsymbol{R}^{n_u\times n_y}$ and n_c denotes the ROC state. Note: $n_c = 0$ leads to the original SOF control law. For more details, we refer the interested reader to [37], [40] and the references therein.

(ROC1). This four–disk control system from [62] can be stabilized with a reduced order controller of order $n_c = 1$.

$$A = \left[\begin{array}{cccccccc|c} -0.161 & -6.004 & -0.58215 & -9.9835 & -0.40727 & -3.982 & 0 & 0 & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0_{8\times 1} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \\ \multicolumn{8}{c}{0_{1\times 8}} & 0_{1\times 1} \end{array}\right], \quad B_1 = \left[\begin{array}{cc} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \multicolumn{2}{c}{0_{1\times 2}} \end{array}\right], \quad B = \left[\begin{array}{cc} & 1 \\ & 0 \\ & 0 \\ & 0 \\ 0_{8\times 1} & 0 \\ & 0 \\ & 0 \\ & 0 \\ I_{1\times 1} & 0_{1\times 1} \end{array}\right],$$

$$C_1 = \left[\begin{array}{cccccccc|c} 0 & 0 & 0 & 0 & 0.00055 & 0.011 & 0.00132 & 0.018 & \multirow{2}{*}{$0_{2\times 1}$} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \end{array}\right], \quad D_{11} = \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array}\right], \quad D_{12} = \left[\begin{array}{cc} 0_{2\times 1} & \begin{array}{c} 0 \\ 1 \end{array} \end{array}\right],$$

$$C = \left[\begin{array}{cccccccc|c} \multicolumn{8}{c}{0_{1\times 8}} & I_{1\times 1} \\ 0 & 0 & 0.0064432 & 0.0023196 & 0.071252 & 1.0002 & 0.10455 & 0.99551 & 0_{1\times 1} \end{array}\right], \quad D_{21} = \left[\begin{array}{cc} \multicolumn{2}{c}{0_{1\times 2}} \\ 0 & 1 \end{array}\right]$$

(ROC2). The third model of the aircraft presented in [26], Case study III 1), p. 1006/1013. The system represents the same airplane as in *(AC7)* at an altitude of 25,500 ft and with a speed of Mach 0.87.

$$A = \left[\begin{array}{ccccccccc|c} -0.00702 & 0.06339 & 0.00518 & -0.55566 & -0.06112 & 0 & 0.00712 & -0.00566 & 0 & \\ -0.01654 & -0.38892 & 1.0057 & 0.00591 & -0.04632 & 0 & 0.01654 & 0.04018 & 0 & \\ 0.00061 & 0.3521 & -0.47381 & 0 & 1.7862 & 0 & -0.00061 & -0.03638 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & -20 & 20 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & -30 & 0 & 0 & 0 & 0_{9\times 1} \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.55454 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.55454 & 0.00555 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.00555 & -0.55454 & \\ \multicolumn{9}{c}{0_{1\times 9}} & 0_{1\times 1} \end{array}\right],$$

$$B_1 = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1.0531 & 0 & 0 & 0 \\ 0 & 1.28981 & 0 & 0 \\ 0 & -54.514 & 0 & 0 \\ \multicolumn{4}{c}{0_{1\times 4}} \end{array}\right], \quad B = \left[\begin{array}{cc} & 0 \\ & 0 \\ & 0 \\ & 0 \\ 0_{9\times 1} & 0 \\ & 30 \\ & 0 \\ & 0 \\ & 0 \\ I_{1\times 1} & 0_{1\times 1} \end{array}\right],$$

$$C_1 = \frac{1}{\sqrt{2}} \left[\begin{array}{cccccccccc} 0.005 & 0.11679 & -0.00172 & 0 & -0.01413 & 0 & -0.005 & -0.01207 & 0 & 0_{1\times 1} \end{array}\right],$$

$$D_{11} = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \end{array}\right], \qquad D_{12} = \left[\begin{array}{cc} 0_{1\times 1} & \frac{1}{\sqrt{2}} \end{array}\right],$$

$$C = \left[\begin{array}{ccccccccc|c} \multicolumn{9}{c}{0_{1\times 9}} & I_{1\times 1} \\ 0.005 & 0.11679 & -0.00172 & 0 & -0.01413 & 0 & -0.005 & -0.01207 & 0 & \multirow{2}{*}{$0_{2\times 1}$} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \end{array}\right], \quad D_{21} = \left[\begin{array}{cccc} \multicolumn{4}{c}{0_{1\times 4}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

(ROC3). The following two–input, two–output system of order nine appears in [50]. It was stabilized with $n_c = 2$.

$$A = \begin{bmatrix} \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{matrix} & 0_{9\times 2} \\ 0_{2\times 9} & 0_{2\times 2} \end{bmatrix}, \; B = \begin{bmatrix} 0_{9\times 2} & \begin{matrix} 0 & -1 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ -1 & 0 \end{matrix} \\ I_{2\times 2} & 0_{2\times 2} \end{bmatrix}, \; C = \begin{bmatrix} 0_{2\times 9} & I_{2\times 2} \\ \begin{matrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{matrix} & 0_{2\times 2} \end{bmatrix}$$

(ROC4). The next instance is an augmented four disk control system (coupled rotating discs [6]). This system can be stabilized by ROC of order $n_c = 1$.

$$A = \begin{bmatrix} \begin{matrix} -0.161 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -6.004 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -0.5822 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -9.9835 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -0.4073 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -3.982 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & 0_{8\times 1} \\ 0_{1\times 8} & 0_{1\times 1} \end{bmatrix}, \; B_1 = \begin{bmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0.0064 & 0 \\ 0.00235 & 0 \\ 0.0713 & 0 \\ 1.002 & 0 \\ 0.1045 & 0 \\ 0.995 & 0 \end{matrix} \\ 0_{1\times 2} \end{bmatrix}, \; B = \begin{bmatrix} 0_{8\times 1} & \begin{matrix} 0 \\ 0 \\ 0.0064 \\ 0.00235 \\ 0.0713 \\ 1.002 \\ 0.1045 \\ 0.995 \end{matrix} \\ I_{1\times 1} & 0_{1\times 1} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} \begin{matrix} 0 & 0 & 0 & 0 & 0.55 & 11 & 1.32 & 18 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & 0_{2\times 1} \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0_{2\times 1} & \begin{matrix} 0 \\ 0 \end{matrix} \end{bmatrix},$$

$$C = \begin{bmatrix} 0_{1\times 8} & I_{1\times 1} \\ \begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & 0_{1\times 1} \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0_{1\times 2} \\ \begin{matrix} 0 & 0 \end{matrix} \end{bmatrix}$$

(ROC5). The next model describes a Free Gyro–Stabilized Mirror System that can be used to stabilize electro–optical sensors which are mounted on vehicles such as aircraft, helicopter and tanks and therefore subjected to vibrations introduced by these platforms. A detailed description of the system design can be found in [11], p. 309.

$$\bar{a} = 0.004, \quad \bar{b} = 0.00128, \quad \bar{c} = 0.00098, \quad \bar{d} = 0.02,$$
$$\bar{e} = 0.0049, \quad \bar{f} = 0.0025, \quad \bar{g} = 0.00125, \quad \bar{l} = 0.0032, \quad \bar{k} = 0.0025,$$
$$\dot{\Theta}_3 = 1, \quad \Psi_1 = 0.011, \quad \Psi_2 = 0.0011, \quad \epsilon = 5 \cdot 10^{-5},$$
$$N_1 = \bar{a} + \bar{b} + \frac{\bar{e} + \bar{g}}{2} + \bar{l}, \quad N_2 = \bar{c} + \frac{\bar{f}}{4} + \bar{l}$$

$$A = \begin{bmatrix} \begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\bar{k}\cdot\dot{\Theta}_3}{N_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{\bar{k}\cdot\dot{\Theta}_3}{N_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\epsilon & 0 \\ 0 & 0 & 0 & 0 & 0 & -\epsilon \end{matrix} & 0_{6\times 1} \\ 0_{1\times 6} & 0_{1\times 1} \end{bmatrix}, \; B_1 = \begin{bmatrix} \begin{matrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \Psi_1 \\ 0 & 0 & \Psi_2 \end{matrix} \\ 0_{1\times 3} \end{bmatrix}, \; B = \begin{bmatrix} 0_{6\times 1} & \begin{matrix} 0 & 0 \\ \frac{1}{N_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{N_2} \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ I_{1\times 1} & 0_{1\times 2} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{matrix} & 0_{2\times 1} \end{bmatrix}, \qquad D_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0_{2\times 1} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \end{bmatrix},$$

$$C = \begin{bmatrix} 0_{1\times 6} & I_{1\times 1} \\ \begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{matrix} & 0_{4\times 1} \end{bmatrix}, \qquad D_{21} = \begin{bmatrix} 0_{1\times 3} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \end{bmatrix}$$

(ROC6). P. Gahinet presented the following system in [23].

$$A = \begin{bmatrix} \begin{matrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & -2 \end{matrix} & 0_{3\times 2} \\ 0_{2\times 3} & 0_{2\times 2} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \\ & 0_{2\times 3} & \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3\times 2} & \begin{matrix} 1 \\ 0 \\ 1 \end{matrix} \\ I_{2\times 2} & 0_{2\times 1} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} \begin{matrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{matrix} & 0_{3\times 2} \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0_{3\times 2} & \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} \end{bmatrix},$$

$$C = \begin{bmatrix} 0_{2\times 3} & I_{2\times 2} \\ \begin{matrix} 0 & -1 & 1 \end{matrix} & 0_{1\times 2} \end{bmatrix}, \quad D_{21} = \begin{bmatrix} & 0_{2\times 3} & \\ 0 & 0 & 1 \end{bmatrix}$$

(ROC7). This flexible actuator example describes a cart fixed to a vertical plane by a linear spring and constrained to move only along one axis. An embedded mass actuator is attached to the center of mass of the cart and can be rotated in the horizontal plane. For further details on the design of the plant see [20].

$$A = \begin{bmatrix} \begin{matrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.02 \\ 0.2 & 0 & 0 & 0 \end{matrix} & 0_{4\times 1} \\ 0_{1\times 4} & 0_{1\times 1} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -0.2 \\ 0_{1\times 1} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{4\times 1} & \begin{matrix} 0 \\ -0.2 \\ 0 \\ 1 \end{matrix} \\ I_{1\times 1} & 0_{1\times 1} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} \begin{matrix} 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & 0_{3\times 1} \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0_{3\times 1} & \begin{matrix} 0 \\ 0 \\ 0.2 \end{matrix} \end{bmatrix},$$

$$C = \begin{bmatrix} 0_{1\times 4} & I_{1\times 1} \\ \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{matrix} & 0_{2\times 1} \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0_{1\times 1} \\ 0 \\ 0 \end{bmatrix}$$

(ROC8). Here we present a three–mass–spring system as discussed by El Ghaoui *et al* in [19]. It consists of three unit masses connected by a linear spring of unit spring constant. The input acts on the left mass, and the position of the right mass is measured. The optimal ROC gain was computed to be $n_c = 3$ for the corresponding augmented system. Particularly, the augmented three–mass–spring system has the plant matrices

$$A = \begin{bmatrix} \begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{matrix} & 0_{6\times 3} \\ 0_{3\times 6} & 0_{3\times 3} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0_{3\times 1} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{6\times 3} & \begin{matrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ I_{3\times 3} & 0_{3\times 1} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} & 0_{7\times 3} \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0_{7\times 3} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{matrix} \end{bmatrix},$$

$$C = \begin{bmatrix} 0_{3\times 6} & I_{3\times 3} \\ \begin{matrix} 0 & 0 & 0 & 0 & 1 & 0 \end{matrix} & 0_{1\times 3} \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0_{3\times 1} \\ 1 \end{bmatrix}$$

(ROC9). An augmented two–mass–spring system adapted from M. Chilali and P. Gahinet [12].

$$A = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & \\ -1 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 1 & 0_{4\times 2} \\ 1 & 0 & -1 & 0 & \\ \multicolumn{4}{c}{0_{2\times 4}} & 0_{2\times 2} \end{array}\right], \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0_{2\times 1} \end{bmatrix}, \quad B = \begin{bmatrix} & 0 \\ & 1 \\ 0_{4\times 2} & 0 \\ & 0 \\ I_{2\times 2} & 0_{2\times 1} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0_{5\times 2} \\ 0 & 0 & 0 & 1 & \\ 0 & 0 & 0 & 0 & \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} & 0 \\ & 0 \\ 0_{5\times 2} & 0 \\ & 0 \\ & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} \multicolumn{4}{c}{0_{2\times 4}} & I_{2\times 2} \\ 0 & 0 & 1 & 0 & 0_{1\times 2} \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0_{2\times 1} \\ 1 \end{bmatrix}$$

(ROC10). This control problem describes an arm–driven inverted pendulum. It is a two–link system consisting of an actuated arm (first link) and a non–actuated pendulum (second link). The main control objective is to maintain the pendulum in the vertical position using the rotation of the arm. For further details see [4].

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \\ 48.9844 & 0 & -48.9844 & 0 & 0 & \\ 0 & 0 & 0 & 0.18494 & 0 & 0_{5\times 1} \\ 0 & 0 & 0 & -50 & 0 & \\ 0 & 0 & -0.5 & 0 & 0 & \\ \multicolumn{5}{c}{0_{1\times 5}} & 0_{1\times 1} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.5 \\ \multicolumn{2}{c}{0_{1\times 2}} \end{bmatrix}, \quad B = \begin{bmatrix} & 0 \\ & 0 \\ 0_{5\times 1} & 50 \\ & 0 \\ & 0 \\ I_1 & 0_{1\times 1} \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0.0036988 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0_{2\times 1} \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0_{2\times 1} & \begin{matrix} 0 \\ 0 \end{matrix} \end{bmatrix},$$

$$C = \begin{bmatrix} \multicolumn{5}{c}{0_{1\times 5}} & I_1 \\ 0 & 0 & 1 & 0 & 0 & \\ 1 & 0 & -1 & 0 & 0 & 0_{3\times 1} \\ 0 & 0 & 0 & 0 & 1 & \end{bmatrix}, \quad D_{21} = \begin{bmatrix} \multicolumn{2}{c}{0_{1\times 2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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